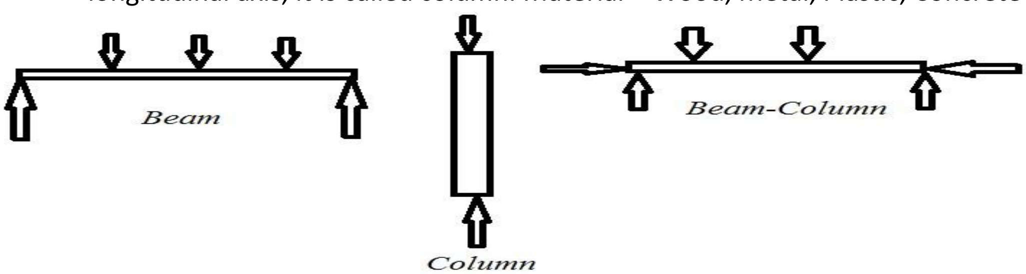
UNIT – II

Shear Force and Bending Moment

Definition of beam – Types of beams – Concept of shear force and bending moment – S.F and B.M diagrams for cantilever, simply supported and overhanging beams subjected to point loads, uniformly distributed load, uniformly varying loads and combination of these loads – Point of contraflexure – Relation between S.F., B.M and rate of loading at a section of a beam.

Definition: Beam

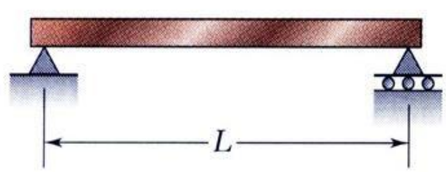
- A bar working under bending is generally termed as beam.
- A beam is laterally (Transverse) loaded member, whose cross-sectional dimensions are small as compared to its length.
- A beam may be defined as a structural member subjected to external loads at right angles to its longitudinal axis. If the external force acts along the longitudinal axis, it is called column. Material – Wood, Metal, Plastic, Concrete



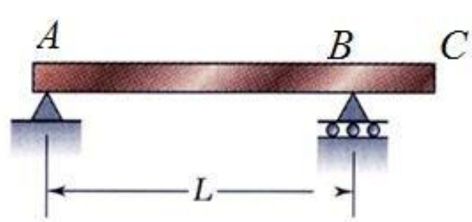
Types of beams: According to their support

1. ***Simply Supported beam***: if their supports creates only the translational constraints.

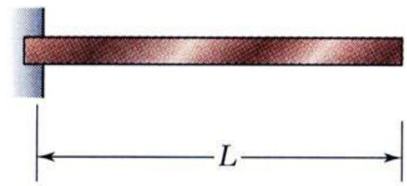
Sometime translational movement may be allowed in one direction with the help of rollers.



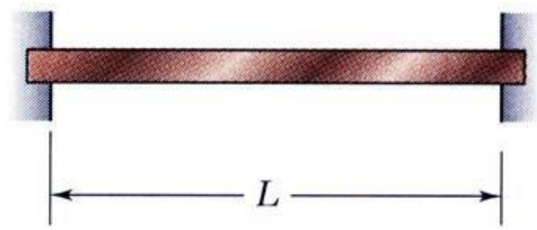
2. ***Overhanging beam***: A beam which is simply supported at point A and B and projects beyond point B. The segment BC is similar to cantilever beam but also the beam axis may rotate at point B.



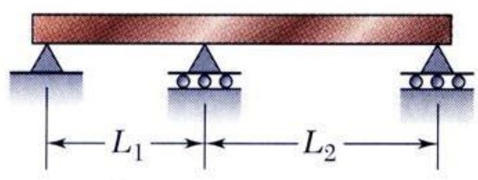
3. ***Cantilever beam:*** fixed at one end and free at other end. At fix support the beam can neither translate nor rotate, whereas at the free end it may do both. Therefore force & movement reactions may exist at the fixed support.



4. ***Fixed beam***: When both end is fixed.



5. ***Continuous beam***: More than two supports are there.



Types of loading

a. ***Concentrated or point load***: When external load acting on the beam is concentrated at a single point on the beam.

b. ***Uniformly distributed load (UDL):*** When external load acting on the beam is distributed over a length of beam, the following load is said to be a UDL. Ex- self-weight of beam, water pressure at bottom of water tank.
It is represented as magnitude of load per unit length. For solving numerical problem, the total UDL is converted into point load, acting at the centre of UDL.

c. ***Uniformly varying load (UVL):*** Load spread over a beam in such a manner that rate of loading varies uniformly from point to point. Also known as triangular load.

Shear Force (SF)

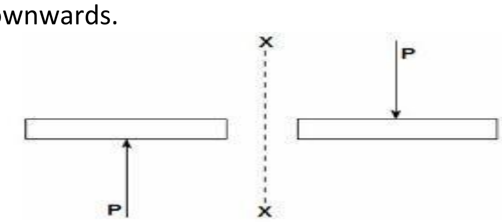
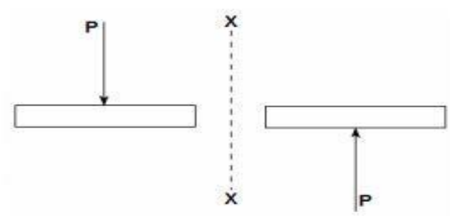
When the beam is loaded in some arbitrarily manner, the internal forces and moments are developed and the term shear force and bending moment come into pictures.

The algebraic sum of vertical forces at any section of beam to the right or left of section is known as Shear Force.

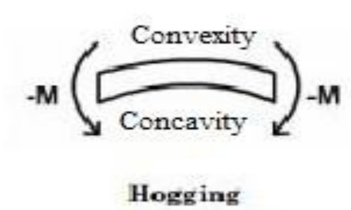
Bending Moment (BM)

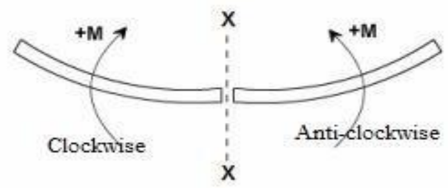
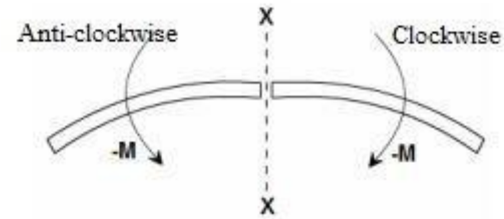
Algebraic sum of moments of all the forces acting to the right or left of the section is known as bending moment.

Sign Convention for Shear Force: Shear Force at section will be considered as

Positive (+)	***Negative (-)***
When resultant of forces to the left to the section is upwards or right of the section is downwards. 	When the left hand portion tend to slide downwards or right hand portion tend to slide upwards 

Sign Convention for Bending Moment:

Positive (+)	***Negative (-)***
We take bending moment as a section as positive, if it tend to bend the beam at that point to a curvature having concavity at top or convexity at the bottom. 	We take bending moment as a section as negative, if it tend to bend the beam at that point to a curvature having convexity at top or concavity at the bottom. 
Called Sagging Bending Moment	Called Hogging Bending Moment

When it is acting in clockwise direction to the left or anti-clockwise direction to the right.	When it is acting in anti-clockwise direction to the left or clockwise direction to the right.
	

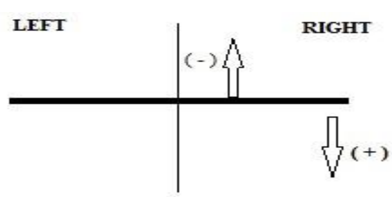
Shear Force Diagrams and Bending Moment Diagrams

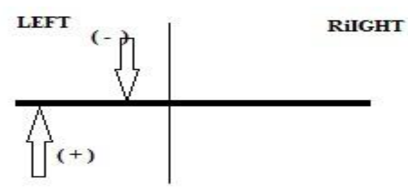
Before taking up the design of any structural element the structure is to be analysed and magnitude of bending moments and shear force should be determined. The structural element is designed for maximum bending moment and for maximum shear force. The BMD help to a great extent in identifying the tensile zones in reinforced concrete structure for providing steel reinforcement at appropriate place.

Important points for drawing SFD & BMD:

1. Consider a left or right portion of section.
2. Add forces (including reactions) normal to the beam on one of the portion.

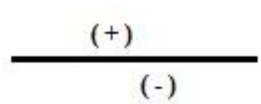
   If right portion of the section is chosen, a force on right portion acting downwards is positive, while force acting upwards is negative.



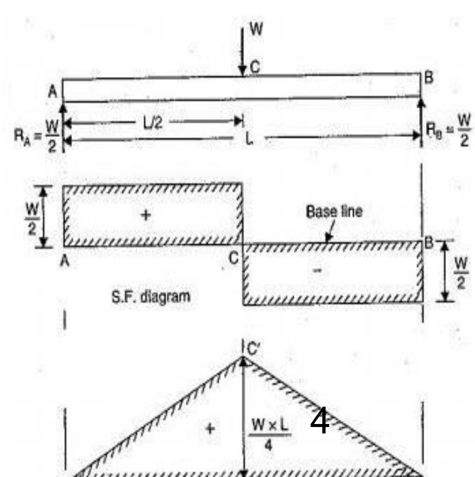


   If left portion of the section is chosen, a force on left portion acting upwards is positive, while force acting downwards is negative.

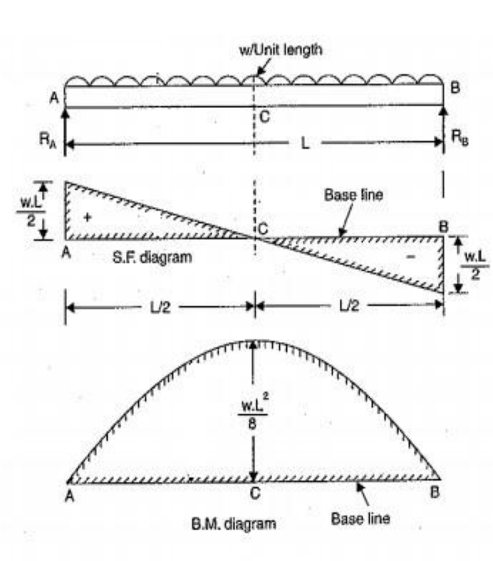
3. The positive value of SF and BM are plotted above the base line and negative value below the base line.

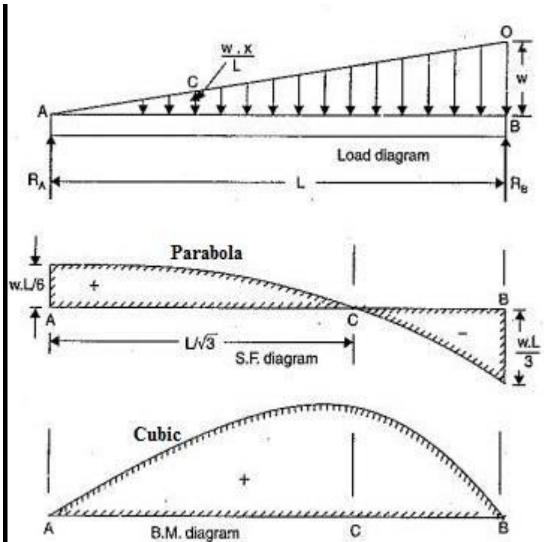


4. The SFD will increase or decrease suddenly i.e. by a vertical straight line at a section where there is a vertical point load but BM remains the same.
5. SF between any two vertical loads will be constant and hence SFD between two vertical loads will be horizontal.

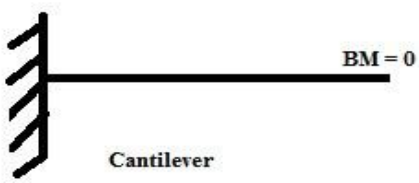


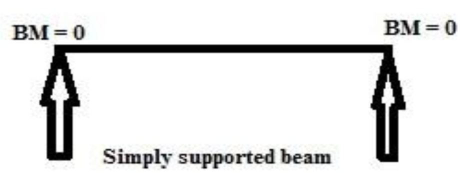
6. If there is no load between two points, then the SF does not change (SF is horizontal) but BM changes linearly (inclined straight line).
7. If there is UDL between two points, then the SF changes linearly (SF is inclined by straight line) but BM changes according to parabolic law (BM will be parabola curve).





8. If there is UVL between two points, then the SF changes parabolic law (SF will be a parabola curve) but BM changes according to cubic law.
9. The BM at the two supports of a simply supported beam and the free end of cantilever will be zero.





10. Bending moment variation is 1 order a head than shear force variation.

Shear Force	**Bending Moment**
Rectangle or Constant	Linear or Triangle
Linear or Triangle	Parabola
Parabola	Cubic

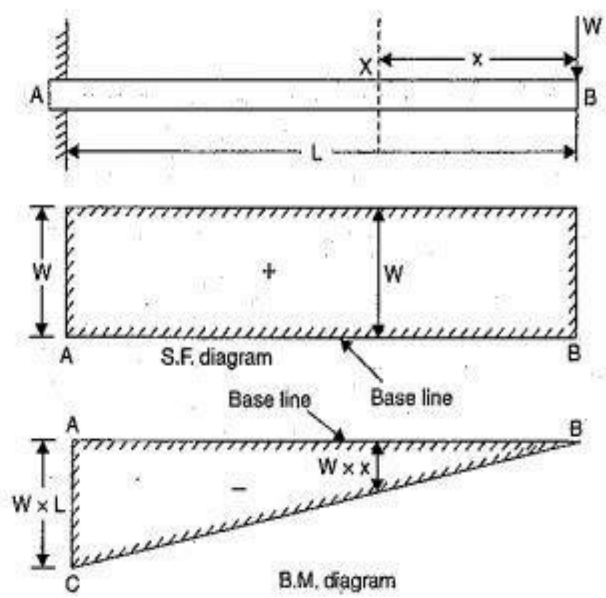
Common Relationships

Load	0	Constant	Linear
Shear	Constant	Linear	Parabolic
Moment	Linear	Parabolic	Cubic

Cantilever beam with a point load at its free end

Consider a cantilever beam AB of length *l*, fixed at A and free at B, and carrying a point load W at the free end B.

Calculation for SFD:



- Take a section *X-X* at a distance of *x* from free end B.
- Consider a right portion of the section.
- The shear force at this section is equal to resultant force acting on the right portion at given section.
- But the resultant force acting on the right portion at the section *X-X* is *W* and acting in downward direction.
- Force on right portion acting downward is considered positive.
- Hence shear force at section *X-X* is positive.
  SF at section *X-X* = +*W*
- There is no other load between A & B.
  So that Shear Force will be constant at all sections of cantilever beam.

Calculation for BMD:

Bending moment at section $X\text{-}X = M_x = -W.x$ ……………………………… *(i)*

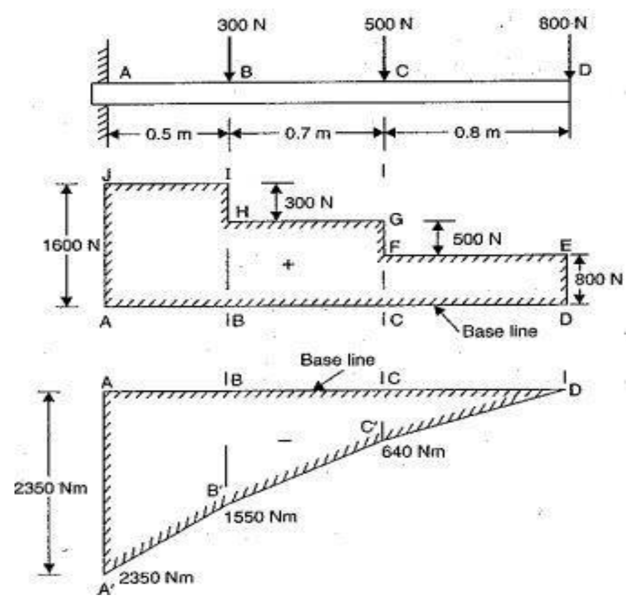
- Bending moment will be negative as for the right portion of the section, the moment of W at X-X is clockwise.
- Bending of cantilever will take place in such a manner that convexity will be at the top of the beam.
- From equation (i) it is clear that BM of a cantilever beam at any section is proportional to the distance of the section from the free end

  BM at point $A_{(x=0)} = 0$

  BM at Point $B_{(x=l)} = -W.l$
- Hence bending moment follows straight line for such cases.

Q. A cantilever beam of length 2m carries the point loads as shown in figure. Draw shear force and bending moment diagram for cantilever beam.

Calculation for SFD:



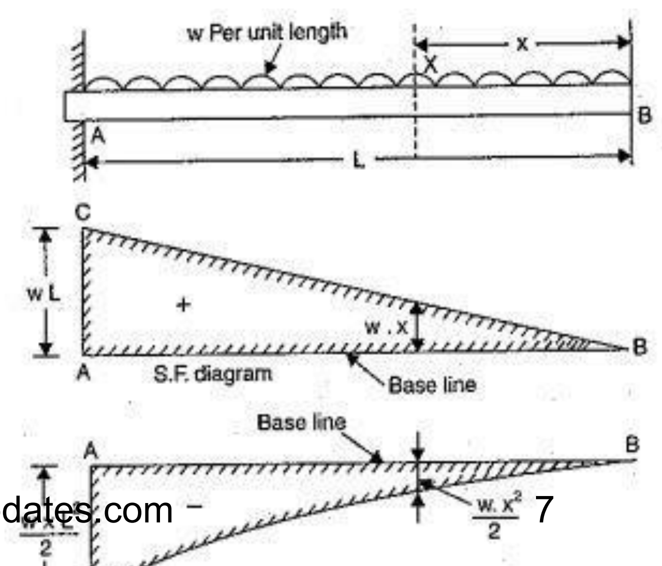
SF at point D = 800N
SF just right to C = 800N
Sf at point C = 800+500 = 1300N
SF just right to B = 1300N
SF at point B = 1300+300 = 1600N
Sf just right to A = 1600N

Calculation for BMD:

BM at point D = 0
BM at point C = −800×0.8 = −640N-m
BM at point B = −800×1.5−500×0.7
= −1550N-m
BM at point A = −800×2−500×1.2−300×0.5
= −2350N-m

Cantilever with Uniformly distributed load

Consider a beam AB of *l* length fixed at A and carrying a uniformly distributed load of *w* per unit length over the entire length of cantilever.



The SF at section *X-X* will be equal to the resultant force acting on the right portion of the section.
Resultant force on right portion = *w.x*

The resultant force is acting downwards.

Calculation for SFD:

SF at section *X-X* = *w.x*
SF at point $B_{(x=0)}$ = 0
SF at point $A_{(x=l)}$ = *w.l*

Calculation for BMD:

As we discussed earlier that the UDL over a section is converted into point load acting at the C.G. of the section

BM at section *X-X* = — $w.x.$ – = —

BM at Point $B_{(x=0)}$ = *0*

BM at point $A_{(x=l)}$ = — —

Q. A cantilever of length 2m carries a uniformly distributed load of 1kN/m run over a length of 1.5m from the free end. Draw the shear force and bending moment diagram for the cantilever.

Calculation for SFD:
SF at section *X-X* between B & C at a distance of x from free end = *w.x = x*
SF at point $B_{(x=0)}$ = 0
SF at point $C_{(x=1.5)}$ = 1.5kN
SF just right to A = 1.5kN

Calculation for BMD:
BM at section *X-X* between B & C at a distance of x from free end = — w.x. – = —
BM at point $B_{(x=0)}$ = *0*

*B*M at point $C_{(x=1.5)}$ = —— = −1.125kN-m

BM at section *Y-Y* between A & C at a distance of *y* from free end

= −1×1.5× [—— + (*x*—1.5)] = —1.5 (*x*—0.75)

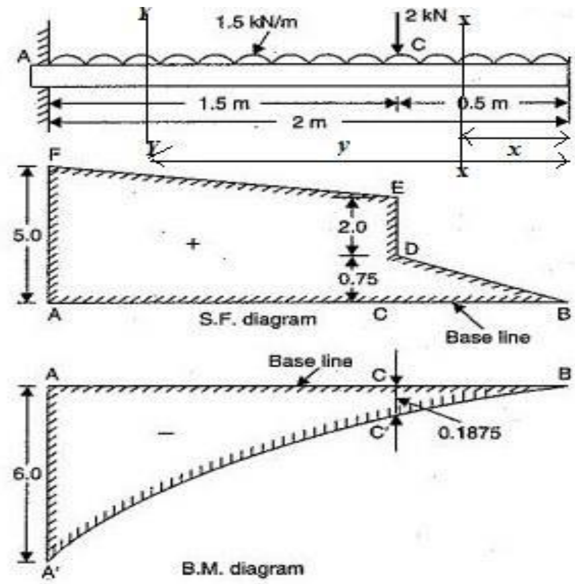
[Total load due to UDL is = 1×1.5 = 1.5kN

This load will act at a distance = —— = 0.75m]

BM at point $C_{(x=1.5)}$ = —1.5 (1.5—0.75)
= —1.125kN-m
BM at point $A_{(x=2)}$ = —1.5 (2—0.75)
=—1.875kN-m

Q. A cantilever of length 2m carries a UDL of 1.5kN/m run over the whole length and a point load of 2kN at a distance of 0.5m from the free end. Draw SF and BM diagram.

Calculation for SFD:



SF at section *X-X* between B & C at a distance of x from free end = *w.x =1.5 x*

SF at point $B_{(x=0)}$ = 0

SF just right to $C_{(x=0.5)}$ = 1.5×0.5 = 0.75kN

BM at any section *Y-Y* between A & C at a distance y from B (free end) is given by

BM at section *Y-Y* = (1.5×*y*+2*)* kN

SF at point $C_{(y=0.5)}$ = 0.75+2 = 2.75kN

SF just right to $A_{(y=2)}$ = 5kN

Calculation for BMD:

BM at section *X-X* between B & C at a distance of x from free end = $-w.x\ .\ -$

$= -1.5\frac{x}{} = -0.75x^2$

BM at point $B_{(x=0)} = 0$

*B*M at point $C_{(x=0.5)}$ = -0.75×0.5^2

= -0.1875kN-m

BM at section *Y-Y* between A & C at a distance of *y* from free end

= $-1.5y.- -2(y-0.5)$

= $-0.75y^2 - 2$

(y—0.5) *B*M at point $C_{(y=0.5)}$

= -0.75×0.5^2

$-2\ (0.5-0.5)$

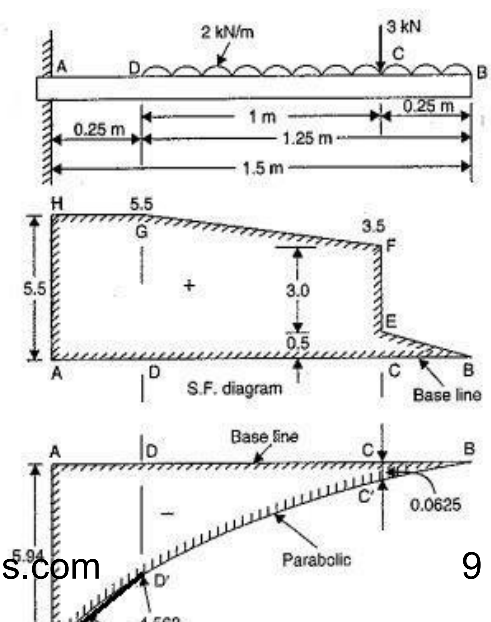
=

-0.1875kN-m

BM at point $A_{(y=2)}$ = $-0.75\times2^2 - 2\ (2-0.5)$

=-6kN-m

Q. A cantilever 1.5m long is loaded with a uniformly distributed load of 2kN/m run over a length of 1.25m from the free end. It also carries a point load of 3kN at a distance of 0.25m from the free end. Draw SFD & BMD of cantilever.



Calculation for SFD:

SF at point B = 0

SF just right to C = 2×0.25 = 0.50kN

SF at point C = 0.50+3 = 3.50kN

SF just right to D = 3.5+2×1 = 5.5kN

SF at point D = 5.5kN

SF just right to A= 5.5kN

Calculation for BMD:

BM at Point B = *0*

BM at point C = −2×0.25× ——

=

−0.0625kN-m

BM at point D =

−2×1.25× —— −3 ×1

= −4.5625kN-m

BM at point A = −2×1.25× [—— + 0.25] −3 × [1+0.25]   = −5.9375kN-m

Q. A cantilever of length of 5m is loaded as shown in figure. Draw SFD and BMD for cantilever.

Calculation for SFD:

SF at point B = 2.5kN

SF just right to C = 2.5kN

SF at point C = 2.5kN

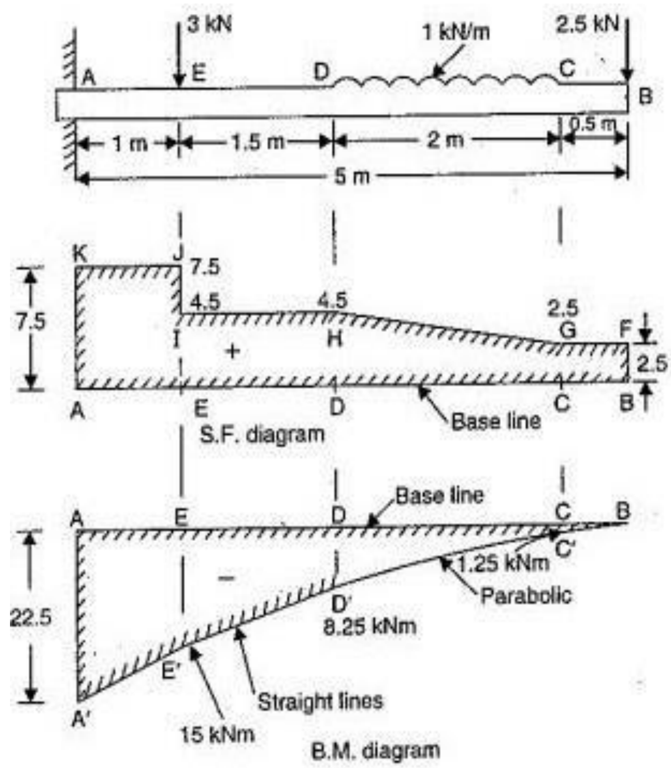
SF just right to D = 2.5+1×2 = 4.5kN

SF at point D = 4.5kN

SF just right to E = 4.5kN

SF at point E= 4.5+3 = 7.5kN

SF just right to A = 7.5kN



Calculation for BMD:

BM at Point B = 0

BM at point C = −2.5×0. 5 = −1.25kN-m

BM at point D = −2.5 × (2+0.5) −1×2×−

= −8.25kN-m

BM at point E = −2.5× [0.5 + 2 +1.5] -1×2×[−+1.5]

= 15kN-m

BM at point A = -2.5×5 – 1×2×[−+2.5] −3×1

= −22.5kN-m

Cantilever with a gradually varying load

A cantilever of length l fixed at A and carried a gradually varying load from zero at free end to w per unit length at fixed end.

Rate of loading is 0 at B and is w per meter run at A

That means rate of loading for a length l is w per unit length

By using similar triangle ABE and CBD

— = — That means — = —

CD = — × x

Rate of loading for a length of x = — × x per unit length

Calculation for SFD:

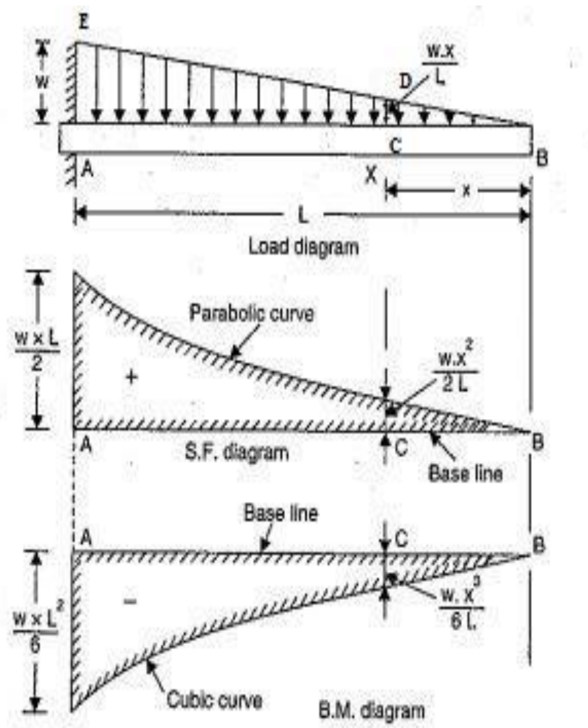
SF at section X-X between A & B at a distance of x from free end = Total load on the cantilever for a length of x from the free end B

= Area of triangle CBD CD = – ×CB×CD =– × ? × (—× x) = ——

SF at point B(x=0) = 0

SF at point A(x=l) = —— = —

Calculation for BMD:



BM at section X-X between A & B at a distance of x from free end
= – (Total load for a length x) × Distance of load from C
= – (Area of Triangle CBD) × Distance of C.G. of triangle from C

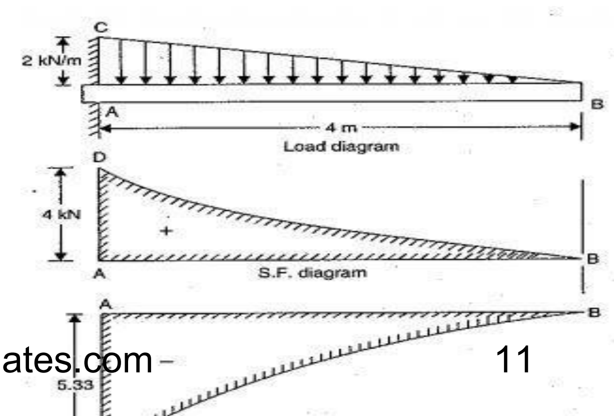
= – —— × –

= – ——

SF at point B(x=0) = 0

SF at point A(x=l) = – —— = – ——

Q. A cantilever of length 4m carries a gradually varying load, 0 at free end to 2kN/m at the fixed end. Draw SFD and BMD for cantilever.



Calculation for SFD:
SF at point B = 0

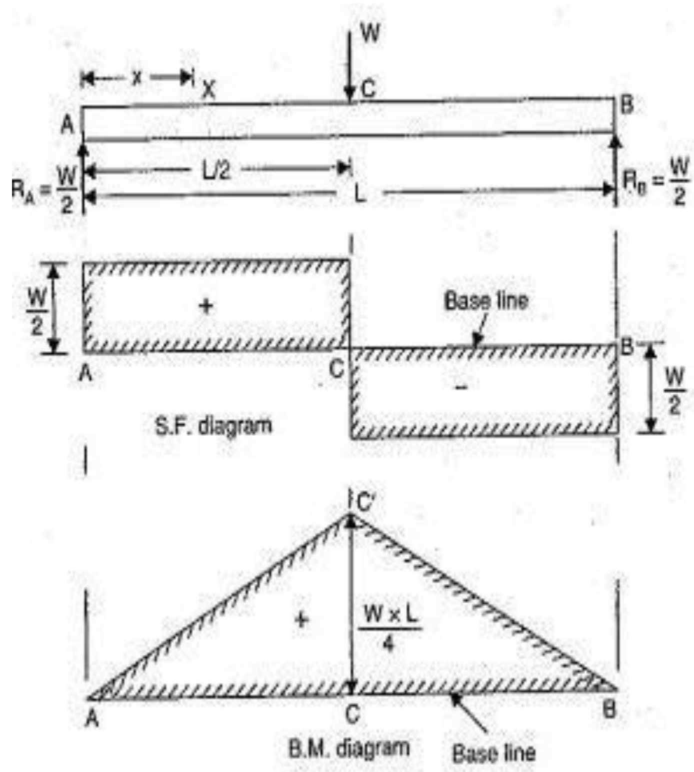
SF just right to A = 4×– = 4 kN

Calculation for BMD: BM at Point B = *0*

BM at point C = —— = — 5.33kN-m

Simply supported beam with a point load at its mid-point

A beam of length *l* simply supported at the ends A and B, carrying a point load W at its middle point C.



Calculation for Reaction:

RA +RB = W

Taking moment at point A, ΣMA = 0

RA×0+RB×l = —

RB = —

RA = —

Calculation for SFD:

SF at point A = —

SF at point C = — – W = —

SF just left to B = – —

Calculation for BMD:

BM at section X-X between A & C at a distance of x from its left end = RA ×x = — ×x

BM at point A(x=0) = 0

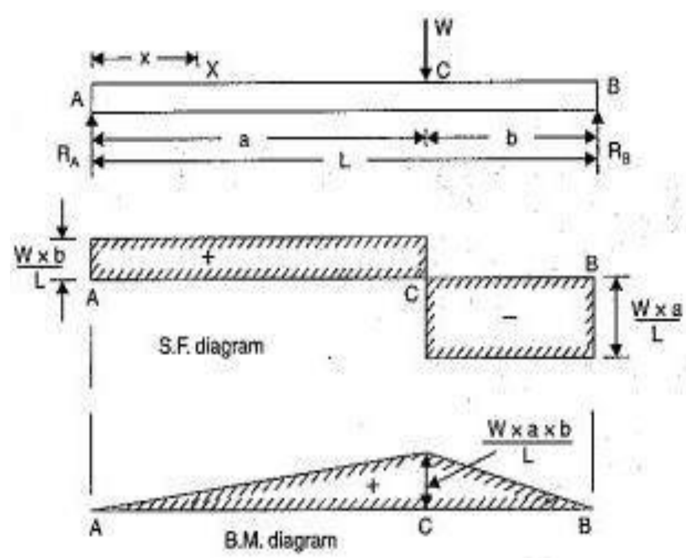
BM at point B(x=0) = — × – = —

BM at section Y-Y between B &C at a distance of y from its left end = R_A×Y – W(*y* – —)

BM at point C(x=l/2) = — ×l/2 = —

BM at point B(x=l) = — ×l–W (l ––) = — – — = 0

Simply supported beam with eccentric point load



Calculation for Reaction:

RA+RB = W

ΣMA=0

RA×0+l×RB = Wa

RB = —

RA = W— = W (l–b)/l = —

Calculation for SFD:

SF at point A = ——

Sf at point C = —— – W = – ——

SF left to B = – ——

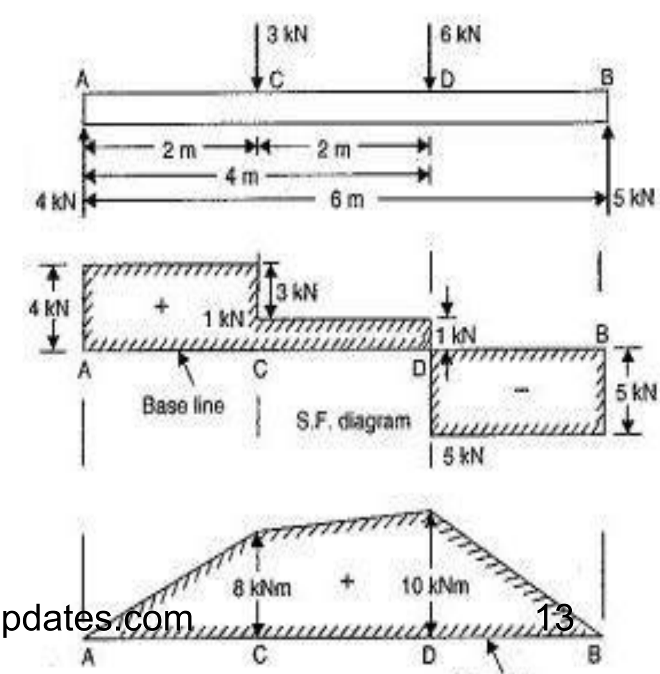
Calculation for BMD:

BM at point A = 0

BM at point C = ——

BM at point B = —— – Wb = 0

Q. A simply supported beam of length 6m carries point load of 3kN and 6kN at distance of 2m and 4m from the left end. Draw SFD and BMD for the beam.



Calculation for Reaction:

RA + RB = 3+6 = 9

ΣMA = 0

RA×0 + RB×6 = 3×2+6×4

6RB = 30

RB = 5kN

RA = 4kN

Calculation for SFD:

SF at point A = 4kN

SF at point C = 4–3 = 1kN

SF at point D = 1−6 = −5kN
SF just left to B = −5kN

Calculation for BMD:
BM at point A = 0
BM at point C = 4*2 = 8 kN-m
BM at point D = 4*4 – 3*2 = 10 kN-m
BM at point B = 4*6 - 3*4 – 6*2 = 0kN-m

Simply Supported beam with a uniformly distributed load

A beam AB of length l simply supported at the end A & B and carrying a uniformly distributed lod of w per unit length over the entire length.

The reaction at their support will be equal and their magnitude will be half the load on the entire length.

Calculation for Reaction:

RA+RB = w.l

ΣMA=0

RA×0+l×RB = w.l. –

RB= —

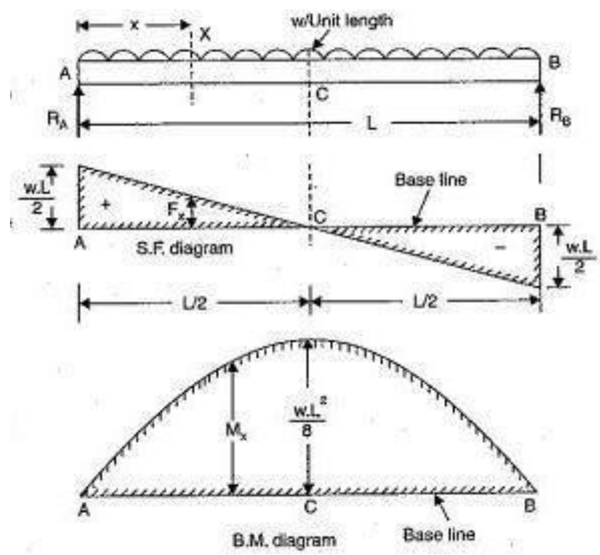
RA = w.l– — = —

Calculation for Shear Force Diagram:

Shear Force at point A = —

Shear Force just left to C = — – w × – = 0

Shear Force just left to B = —



Calculation for Bending Moment Diagram:

BM at point A = RA×0 = 0

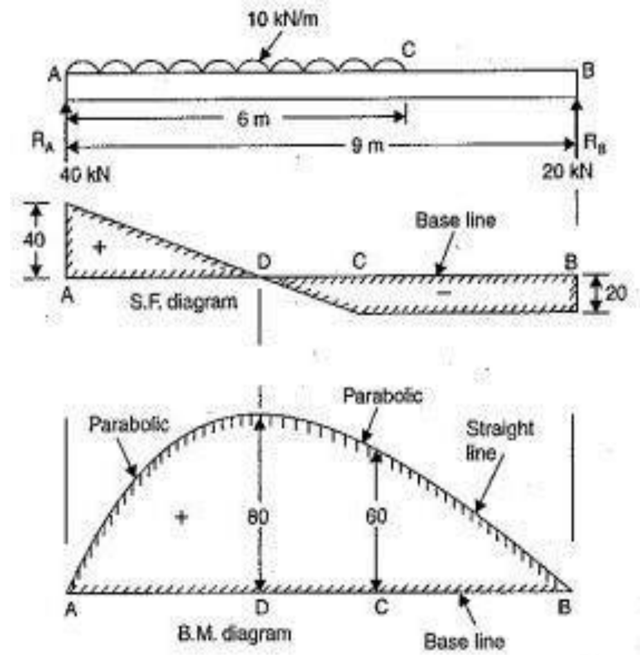
BM at point C = RA×– — w×– ×—

= —× – —

= —

BM at point B = —×––w ×–×– = 0

Q. Draw Shear Force Diagram and Bending Moment Diagram for a simply supported beam of length 9m and carrying a uniformly distributed load of a 10kN/m for a distance of 4m as shown in figure.



Calculation for Reaction:

RA+RB = 10×4 = 40

ΣMA=0

RA×0+9×RB = 10× 6 ×–

9×RB = 180

RB = 20 kN

RA = 40 kN

Calculation for Shear Force Diagram:

Shear Force at point A = 40 kN

Shear Force just left to C = 40 − 10 × 6

= −20 kN

Shear Force at Point C = −20 kN

Shear Force just left to B = −20 kN

SF at A is +40 kN & at C = −20 kN

SF between A to C varies by a straight line

This means somewhere between A to C, the SF is Zero

Let SF is Zero at a distance of x from A

Substitute SF value at this section equal to Zero

Shear Force at section X-X between A & C = 40 − 10×x = 0

x = 4m

Hence SF will be Zero at a distance of x= 4m from A

Calculation for Bending Moment Diagram:

BM at point A = RA×0 = 0

BM at point C = 40× 6 − 10 × 6 ×–

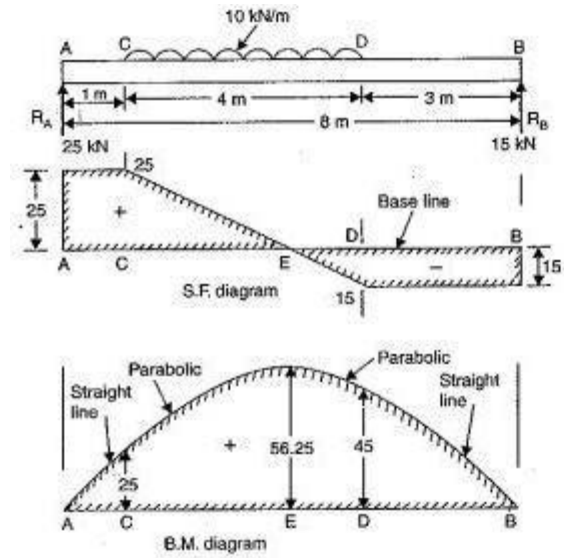
= 240 − 180 = 60 kN-m

BM at point B = 40 × 9 − 10 × 6 × (− + 3) = 0

We know BM will be Maximum where SF changes its sign i.e. SF is Zero.

Maximum Bending Moment = 40× 4 − 10 × 4 × − = 80 kN-m

Q. Draw the Shear Force Diagram and Bending Moment Diagram for a simply supported beam of length 8m and carrying a uniformly distributed load of 10 kN/m for a distance of 4m as shown in figure.



Calculation for Reaction:

RA+RB = 10×4 = 40
ΣMA=0
RARA×0+8×RB = 10× 4 × (−
8×RB = 120
RB = 15 kN
RA = 25 kN

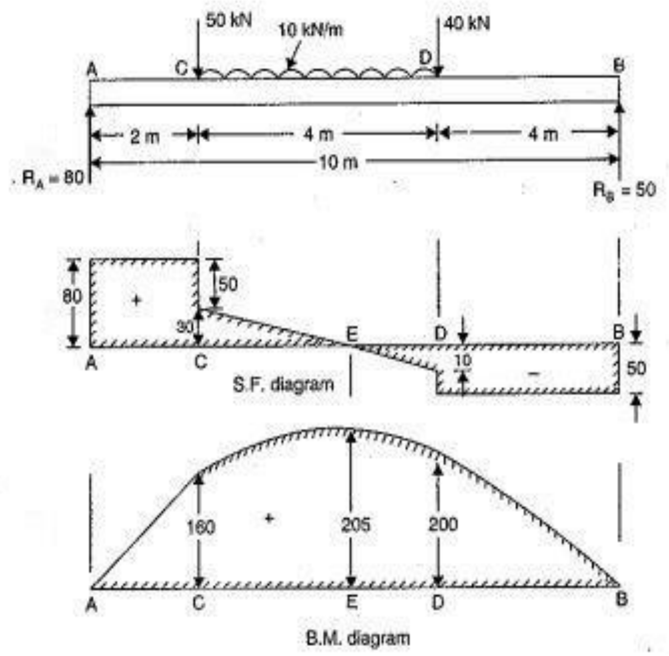
Calculation for Shear Force Diagram:

Shear Force at point A = 25 kN
Shear Force at Point C = 25 kN
Shear Force at point D = 25 − 10 × 4 = -15kN
Shear Force just left to B = -15 kN
Shear Force at C = +25 kN and Shear Force at D = - 15 kN
That means somewhere between C & D, Shear Force is Zero
Shear Force at section X-X between C & D = 25 - 10× (□ − 1) = 0
x = 3.5m
Shear Force at section X-X will be Zero.

Calculation for Bending Moment Diagram:

BM at point A = RA×0 = 0
BM at point C =25× 1 = 25 kN-m
BM at point D=$25 \times 5 - 10 \times 4 \times -$ = 45 kN-m
BM at point B = 25 × 8 - 10 × 4 × (−+ 3) = 0 kN-m
Maximum Bending Moment (at x = 3.5m)-m =25 × 3.5 - 10 × 2.5 × —— = 56.25 kN-m

Q. A simply supported beam of length 10m carries the uniformly distributed load and two points loads as shown in figure. Draw Shear Force Diagram and Bending Moment Diagram for the beam. Also Calculate maximum bending moment of the beam.



Calculation for Reaction:

RA+RB = 50 + 40 + 10×4 = 130 kN

ΣMA=0

RA×0+10×RB = 50 × 2 + 10× 4 × (− + 2)

+ 40 × 6

10×RB = 100 + 240 +160

RB = 50kN

RA = 80 kN

Calculation for Shear Force Diagram:

Shear Force at point A = 80 kN

Shear Force at Point C = 80 – 50 = 30 kN Shear Force just left to D = 30 − 10 × 4

= −10 kN

Shear Force at point D = −10 − 40

= −50 kN

Shear Force just left to B = −50 kN

SF at section X-X between C & D = 80 – 50 − 10(x−2) = 0

30 − 10(x−2) = 0▯ = 5m

Calculation for Bending Moment Diagram:

BM at point A = RA×0 = 0

BM at point C =80 × 2 = 160 kN-m

BM at point D = 80 × 6 − 50 × 4 − 10 × 4 ×− = 200 kN-m

BM at point B = 80 × 10 − 50 × 8 − 10 × 4 × (−+ 4) − 40 × 4 = 0 kN-m

We know that Maximum bending moment will occur at that point where SF changes its sign

(SF = 0)

Maximum Bending Moment (at x = 5m) = 80 × 5 − 50 × 3 − 10 × 3 × − = 205 kN-m

Simply supported beam carrying a uniformly varying load (Triangle load)

(from zero at both ends to w per unit length at the centre)

Consider a simply supported beam AB of span *l* and carrying a triangular load, varying gradually from 0 at both ends to w per unit length at the centre.

Since the load is symmetrical therefore the reaction R_A & R_B will be equal.

Total load on beam = Area of load diagram

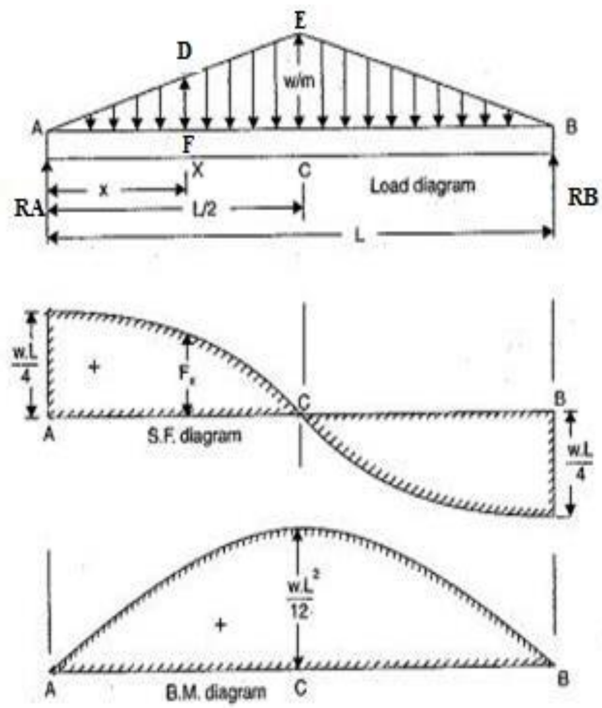
ABE

= − × $base$ × height = −× AB × CE

$= \frac{wl}{2}$

Hence $R_A = R_B$ = Half of the total load

$R_A = \frac{\quad}{4}$ & R_B = —



Consider any section X-X between A & C at a distance x from A.

Rate of loading at section X-X = DF

ΔACE & Δ AFD is similar triangle, So

Load on length AF of the beam = Area of load diagram AFD

= − × $base$ × height = −× AF × DF

$= \frac{1}{2} \times x \times \frac{2wx}{l} = \frac{wx^2}{l}$

This load will act at a distance of $\frac{x}{3}$ from DF

Calculation for SFD:

SF at section X-X = R_A – load on length AF

{It showing parabolic equation}

SF at point $A_{(x=0)}$ = —

SF at point $C_{(x=-)}$ = — – —– = 0

SF just left to $B_{)}$ = – —

Calculation for BMD:

BM at section X-X = $R_A \times x$ – load on length AF×

$$= \frac{Wl}{4} \times x - \frac{wx^2}{l} \times \frac{x}{3}$$

= —– —– {It showing cubic equation}

BM at point $A_{(x=0)}$ = 0

BM at point $C_{(x=-)}$ = — × $\frac{l}{-}$ – $\frac{w}{-}$ × $(\frac{l}{-})^3$= 0

$$= \frac{wl^2}{8} - \frac{wl^2}{24} = \frac{wl^2}{12}$$

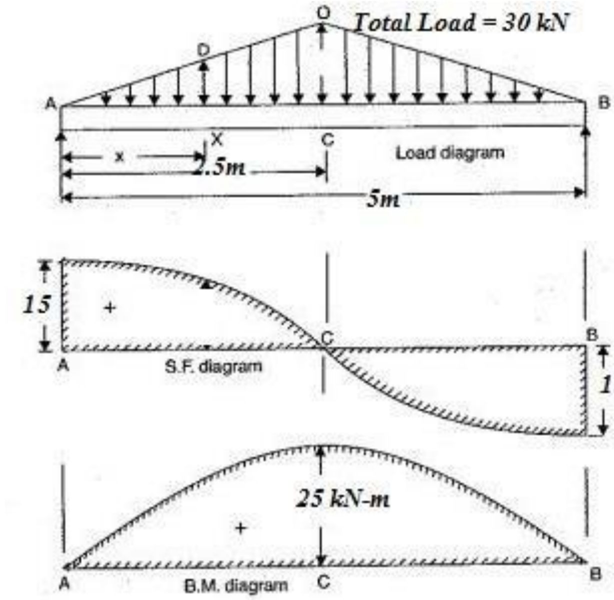
= — {where W = $\frac{Wl}{2}$ }

BM just left to B = 0

NOTE *– Bending moment is always maximum, where Shear Force become zero after changing its sign*

NOTE *– In case of simply supported beam, bending moment will always be zero at both ends (support)*

Q. A simply supported beam of 5m span carries a triangular load of 30kN, Draw SFD and BMD for the beam.



Calculation for reaction:

$R_A = R_B$ = Half of the total load

$R_A = R_B$ = 15 kN

Calculation for SFD:

SF at point A = 15 kN

SF at point C = 15 $-(30/2) = 0$

SF just left to B = — 15 kN

Calculation for BMD:

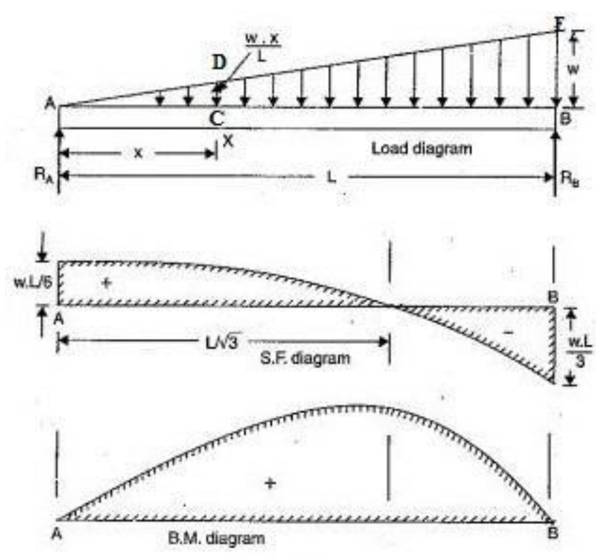
BM at point A = $R_A \times 0 = 0$

BM at point C =15×2.5−1 25 kN-m

BM at point B = 15×5− (3 × 5 = 0kN-m

Simply Supported beam with a gradually varying load from zero at one end to w per unit length at other end

Consider a simply supported beam AB of length l and carrying a gradually varying load from 0 at A to w per unit length at B



Calculation for reaction:

Total load on beam = Area of triangle ABE

= $-\times$ *base* $\times$ height

= $\frac{1}{-} \times w \times l$

R_A+R_B = —

ΣM_A=0

$R_A \times 0 + l \times R_B$ = — × —

R_B = —

R_A = —

Consider any section X-X between A & B at a distance x from A

Rate of loading at section X-X = DC

ΔABE and ΔACD are two similar triangles

— — – — —

Calculation for SFD:

SF at section X-X = R_A – load on length AC

$$= \frac{Wl}{6} - \frac{1}{2} \times x \times \frac{wx}{l}$$ {It showing parabolic equation}

$$= \frac{Wl}{6} - \frac{wx^2}{2l}$$

SF at point A*(x=0)* = —

SF just left to B = — – — = – —

We know that Maximum bending moment will occur at that point where SF changes its sign (SF = 0)

SF at section X-X between A & B at a distance of *x* from A

$\frac{Wl}{6} - \frac{wx^2}{2l} = 0$ $\quad x = \frac{l}{\sqrt{3}}$

Or x = 0.577 l

Calculation for BMD:

BM at section X-X = $R_A \times x$ – load on length AC×

$$= \frac{Wl}{6} \times x - \frac{1}{2} \times x \times \frac{wx}{l} \times \frac{x}{3}$$

$$= \frac{Wlx}{6} - \frac{wx^3}{6l}$$ {It showing cubic equation}

BM at point $A_{(x=0)}$ = 0

BM at point C = — – — = 0

Calculation for Maximum Bending Moment:

BM will be maximum at that point where Shear Force is zero, i.e. at *x* = —

Maximum bending moment $\left(\text{at} = \frac{l}{\sqrt{3}}\right) = \frac{Wl}{6} \times \frac{l}{\sqrt{3}} - \frac{w}{6l} \times \left[\frac{l}{\sqrt{3}}\right]^3 = \frac{wl^2}{9\sqrt{3}}$

Q. A simply supported beam of length 5m carries a uniformly increasing load of 800N/m run at one end to 1600N/m run at other end. Draw the shear force and bending moment diagram for the beam, also calculate the position & magnitude of maximum bending moment.

Weight may be assumed split into

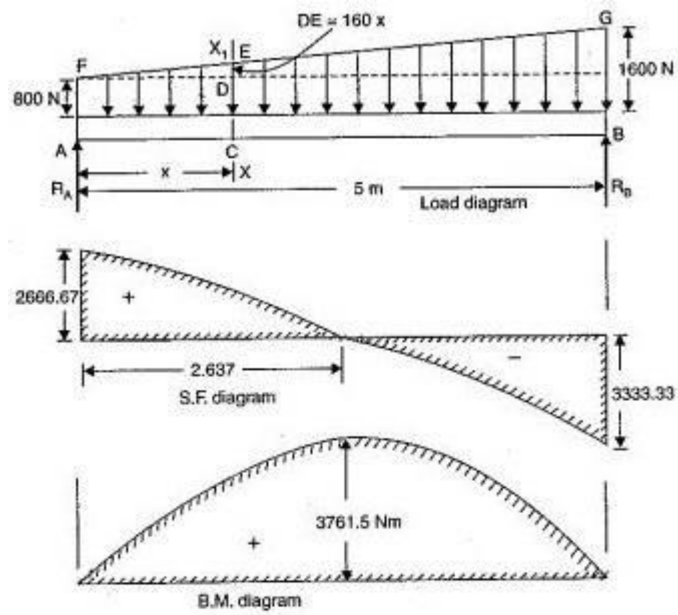
a. Uniformly distributed load of 800N/m over the entire span
b. Uniformly gradually varying load of 0 at C & 800N/m at B

Total UDL = W_{UDL} = $w \times l$ = 800 × 5 = 4000N

Total UVL = W_{UVL} = $\frac{}{2}$ × × height = $\frac{1}{}$ × 5 × 800 = 2000N

Total load acting on beam = W_{UDL} + W_{UVL} = 4000+2000 = 6000N

ΔCDE and ΔCGH are two similar triangles



$\frac{ED}{CD} = \frac{HG}{CC}$ $\frac{800}{5} = \frac{HG}{x}$

HG = 160

Calculation for reaction:

$R_A + R_B$ = 6000N

ΣM_A=0

$R_A \times 0 + 5 \times R_B$ = 800× 5 ×$\frac{5}{}$ + 5 × ($\frac{1}{}$ × 5 × 800) × (——)

$5R_B$ = 10000+6666.66

$6 \times R_B$ = 54

R_B = 3333.33N

R_A = 2666.67N

Calculation for SFD:

SF at section X-X between A & B at a distance of x from A

= R_A – load on length AF

= R_A – (Area of rectangle + Area of ΔCGH)

$$= 2666.67 - 800\times x - (\frac{1}{2} \times x \times 160x)$$

= 2666.67 – 800x 80 ^{2}

SF at point $A_{(x=0)}$ = 2666.67 N

SF at point B $_{(x=5)}$ =2666.67 – 800× 80× 5^2 = − 3333.33N

We know that, Shear Force changes its sign from positive to negative, that means at somewhere there will be a point where Shear will be Zero. And at this point Bending Moment will be maximum.

2666.67 – 800x 80 x^2 = 0

x = 2.637m

Calculation for BMD:

BM at section X-X between A & B at a distance of *x* from A

$$= R_A \times x - 800 \times x \times \frac{x}{2} - \left(\frac{1}{2} \times x \times 160x\right) \times \frac{x}{3}$$

$$= 2666.67\,x - 400x^2 - \frac{80x^3}{3}$$

BM at point A (x=0) = 0

BM at point B (x=5) = $2666.67 \times 5 - 400 \times 5^2 -$ ——

Maximum Bending Moment (x=2.637) = $2666.67 \times 2.637 - 400 \times 2.637^2 -$ ————

Maximum Bending Moment (x=2.637) = 3761.5 kN-m

Point of Contraflexure (or Point of Inflexion):

Bending moment in cantilever was negative, whereas that in a simply supported beam is positive. It is thus obvious that in an overhanging beam, there will be a point where Bending moment will change sign from negative to positive or vice-versa, such a point, where the bending moment changes sign is known as Point of contraflexure or point of inflexion.

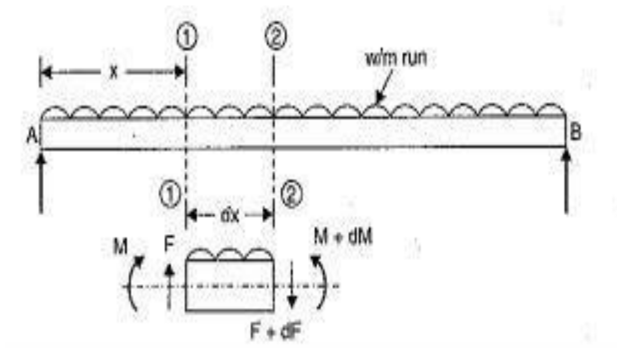
- The point of contraflexure is the point where the bending moment changes its sign from positive to negative or bending moment from Sagging to Hogging and vice-versa.
- At point of contraflexure bending moment is zero.
- It is to be noted that all the points where BM is zero are not necessary point of contraflexure.

Shear Force and Bending Moment Diagram for overhanging beams:

If the end portion of the beam is extended beyond the support, such beam is known as overhanging beam. Overhanging can be assumed as a combination of simply supported beam and cantilever beam. Due to this, overhanging beam experienced point of contraflexure.

Relationship between Load, Shear Force and Bending Moment

Consider a simply supported beam carrying uniformly distributed load of *w* per unit length. An element *dx* at a distance of *x* from the left end A is considered.



An equilibrium of the portion of beam between sectoin1-1 and section 2-2 *F* = Shear Force at section 1-1

F + dF = Shear Force at section 2-2

M = Bending moment at section 1-1

M + dM = Bending moment at section 2-2

Total load on length dx of beam = *w.dx*

For equilibrium ΣV = 0

$F - w.dx = F + dF$

$-w.dx = dF$

$\frac{dF}{dx} = -w$

This shows that the rate of change of Shear Force is equal to load.

Taking moment at section 2-2 i.e. at point D

$M + F.dx - w.dx.dx/2 = M + dM$

Neglecting higher power of small quantities

$F.dx - dM = 0$

F = —

This relationship shows that the rate of change of bending moment is equal to shear force.

– w = —

– w = — . — = —

w = — = —